Principles of Slow Release Relay Design

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This article presents an analytical treatment of the relations controlling the release time of slow release relays, in which field decay is delayed by the currents induced in a conducting sleeve or slug. An hyperbolic relation between flux and magnetomotive force, fitting the decreasing magnetization curve, is used for the relation between induced voltage and current in the sleeve circuit in determining the rate of field decay. Methods are given for estimating and measuring the magnetization constants and those appearing in the relation between pull and field flux.

These relations are used as a basis for a discussion of the design of slow release relays and of the adjustment procedures employed to meet timing requirements.

1 INTRODUCTION

Slow release relays are built and adjusted to provide a time delay between the opening of the coil circuit and the release motion which restores the contacts to their unoperated condition. They are used to assure a desired sequence of circuit operation, as, for example, in maintaining a closed path through the slow release relay's contacts during the pulses sent in dialing a digit, and opening this path during the much longer interval between digits. Slow release relays constitute a minor but significant part of the relay population in an automatic central office: about ten per cent of the total.

For economy in manufacture, installation, and use, slow release relays are made as similar to the ordinary or general purpose relays with which they are used as their special requirements permit. Each general structural relay developed for telephone work has had a variant form for slow release use. Thus the Y type[1] relay is the slow release form of the U type relay[2] widely used in the Bell System, while the AG relay is the slow release variant of the recently developed wire spring[3] relay.

The circuit functions of most slow release relays permit considerable variation in release time if the minimum delay specified is assured. This

tolerance in the timing requirements permits the use of more economical practices in the construction and use of slow release relays than would be needed for closer control. As these wide tolerances apply to the great majority of applications, over-all economy is attained by developing slow release relays with reference to them, using other devices for the special applications requiring close timing control.

The magnitude of time delay desired is fixed by the operate and release times of the associated general purpose relays. The latter times lie in the range from 5 to 50 milliseconds, with the majority in the lower part of the range. The delays required to assure sequences of events each requiring time intervals of this order consequently cover the range from 50 to 500 milliseconds (one twentieth to one half a second). Delays of less than 100 milliseconds can generally be provided by ordinary general purpose relays having shorted secondary windings or sleeves (single turn conductors) to delay their release. Special slow release relays are used for delays in excess of 100 milliseconds.

Factors Controlling Release Delay

In terms of circuit operation, release time is the interval from the opening of the coil circuit to the completion of contact actuation during the return motion of the armature. This time is the sum of (a) the time for the magnetic field to decay to the level at which the pull just equals the operated spring load and (b) the motion time for contact actuation. The motion time is never more than a few milliseconds, and is therefore a trivial increment to the long delays of slow release relays. The release time of such relays is therefore, for practical purposes, simply the time of field decay.

When the coil circuit is opened, the field decay induces currents in any circuit linking the field. These currents tend to maintain the field and to delay its decay. A short circuited winding or the single high conductivity turn provided by a copper sleeve gives a high magnetomotive force for a low induced voltage, resulting in a relatively slow decline in field strength. The time for the flux to reach a given level depends upon the conductance of the sleeve, and upon the reluctance of the electromagnet: the ratio of magnetomotive force to flux. The flux level which determines the end of the delay is that for which the pull equals the spring load. The delay is therefore prolonged by a pull characteristic such that the load is held until the flux drops to a minor fraction of its initial value.

Thus the essential features of a slow release relay are a shorted winding

or sleeve, a low reluctance magnetic circuit, and a high level of pull for relatively low values of field strength. In the following analysis of slow release performance, expressions are developed for the time of field decay for the decreasing magnetization characteristic, for the reluctance of the electromagnet, and for its pull characteristics. These expressions permit the estimation of release times, indicate the design conditions to be satisfied to attain a desired level of delay, and indicate the effect on the release time of variations in the spring load, in the dimensions of the electromagnet, and in other design parameters.

The notation used in this article conforms to the list that is given on page 257.

2 FIELD DECAY RELATIONS

If a closed circuit of resistance R and N turns links a magnetic field of flux φ, the voltage equation is:

$$iR + N\frac{d\varphi}{dt} = 0,$$

where i is the circuit current. Multiplying by $4\pi N$ and dividing by R this equation may be written as:

$$\mathfrak{F}_i + 4\pi G_i \frac{d\varphi}{dt} = 0,$$

where $\mathfrak{F}_i$ is the magnetomotive force of this circuit, and G_i is its value of N^2/R, which may be termed the equivalent single turn conductance. If there are several such circuits linking the same magnetic field, a similar expression applies to each, and these may be added to give the equation:

$$\mathfrak{F} + 4\pi G \frac{d\varphi}{dt} = 0, \tag{1}$$

where $\mathfrak{F} = \sum \mathfrak{F}_i$, and $G = \sum G_i$. In the case of a slow release relay, one linking circuit is usually a sleeve, whose conductance may be designated G_S. The applications are identical for a short circuited winding, if the applicable value of N^2/R is substituted for G_S. In either case, G also includes a term G_E representing the net effect of the eddy current paths. As the eddy currents at different distances from the center of the core link different fractions of the total field, this representation by a single term is an approximation. As shown in a companion article,[4] however, the approximation is satisfactory when G_E is a minor part of G, as in the

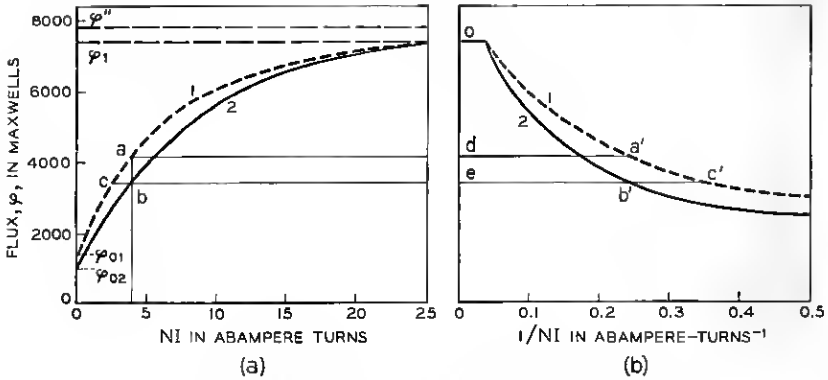

Fig. 2 — Graphical determination of release time.

explicit a statement of the general relations applying as does the approximate analytical treatment outlined below.

Hyperbolic Approximation to Decreasing Magnetization Curve

The decreasing magnetization curve, as illustrated in Fig. 1, has the general character of a rectangular hyperbola, and may therefore be represented approximately by the empirical equation:

$$\frac{\mathfrak{F} + \mathfrak{F}_C}{\varphi} = \frac{\mathfrak{R}''\varphi''}{\varphi'' - \varphi}, \tag{3}$$

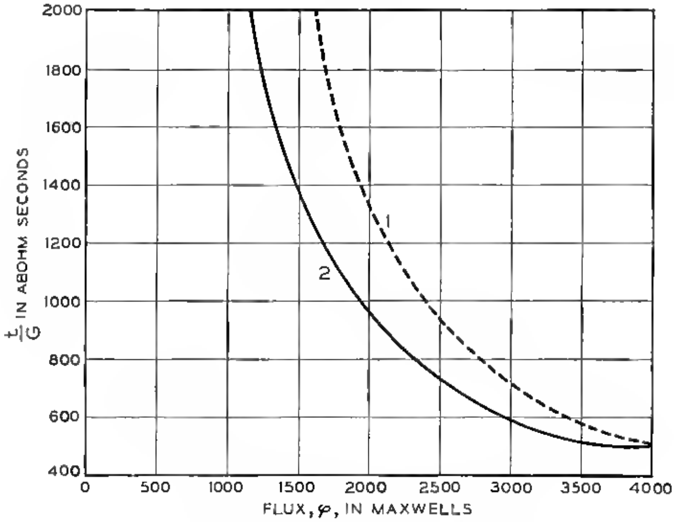

Fig. 3 — Release time versus flux.

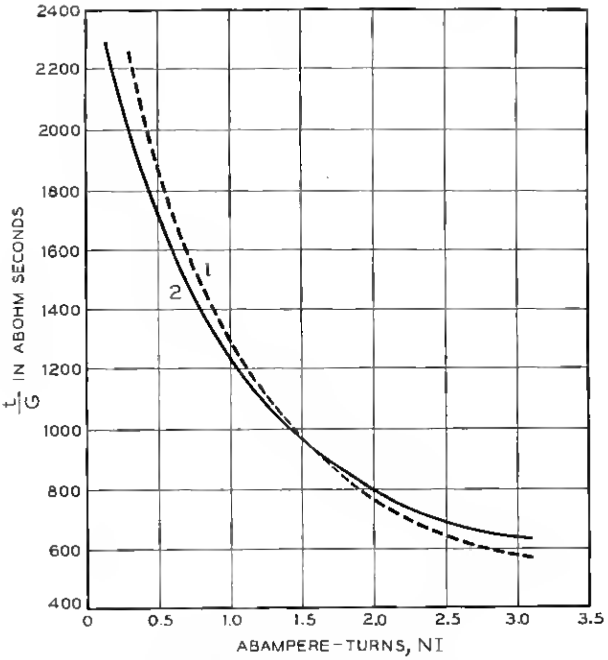

Fig. 4 — Release time versus ampere turns.

for which $\varphi = 0$ for $\mathfrak{F} + \mathfrak{F}_C = 0$, while φ'' is the asymptote approached by φ as $\mathfrak{F}$ is increased. $\mathfrak{R}''$ is the initial incremental reluctance, the value of $d\mathfrak{F}/d\varphi$ for $\varphi = 0$. If φ_0, as in Fig. 1, is the value of φ for $\mathfrak{F} = 0$, the resulting expression for $\mathfrak{F}_C$ may be substituted in (3) and this equation written in the alternative form:

$$\frac{\mathfrak{F}}{\mathfrak{R}''\varphi''} = \frac{\varphi}{\varphi'' - \varphi} - \frac{\varphi_0}{\varphi'' - \varphi_0}. \tag{4}$$

The incremental reluctance $\mathfrak{R}_i$, the value of $d\mathfrak{F}/d\varphi$ at $\varphi = \varphi_0$, is given by:

$$\mathfrak{R}_i = \left(\frac{\varphi''}{\varphi'' - \varphi_0}\right)^2 \mathfrak{R}''. \tag{5}$$

The expression for $\mathfrak{F}/(4\pi)$ given by (4) may be substituted for Ni in (2) to give an expression for the release time t. After clearing fractions, the resulting equation is:

$$\frac{t}{G} = \frac{4\pi(\varphi'' - \varphi_0)^2}{\mathfrak{R}''\varphi''^2} \int_{\varphi}^{\varphi_1} \left(\frac{1}{\varphi'' - \varphi_0} - \frac{1}{\varphi - \varphi_0}\right) d\varphi.$$

From (5), the term outside the integral sign is $4\pi/\mathfrak{R}_i$. On integration, the following expression is obtained for the release time t:

$$t = \frac{4\pi G}{\mathfrak{R}_i}\left(\frac{\varphi - \varphi_1}{\varphi'' - \varphi_0} + \ln\frac{\varphi_1 - \varphi_0}{\varphi - \varphi_0}\right).$$

In slow release relays, a "soak" or high ampere turn value is applied in operation, and the initial value φ_1 is close to the saturation value φ''. It is therefore a satisfactory approximation to take $\varphi_1 = \varphi''$ in the preceding equation, which then reduces to:

$$t = \frac{4\pi G}{\mathfrak{R}_i}\left(\ln z - 1 + \frac{1}{z}\right), \tag{6}$$

where,

$$z = \frac{\varphi'' - \varphi_0}{\varphi - \varphi_0}. \tag{7}$$

The time therefore varies as the bracketed term in (6), which is shown plotted against z in Fig. 5. As z varies inversely as $\varphi - \varphi_0$, the difference between the flux and its ultimate value, (6) gives the release time for the value of φ at which the pull equals the operated load.

To obtain an expression for the release time in terms of the ampere turn value at which release occurs requires an expression for z in terms of $\mathfrak{F}$, or $4\pi NI$. This may be obtained by substituting in (4) the expression for $\mathfrak{R}''$ given by (5). The resulting equation reduces to:

$$\mathfrak{F} = (\varphi'' - \varphi_0)\mathfrak{R}_i\,\frac{\varphi - \varphi_0}{\varphi'' - \varphi} = \frac{(\varphi'' - \varphi_0)\mathfrak{R}_i}{z - 1},$$

giving the equation:

$$z = 1 + \frac{(\varphi'' - \varphi_0)\mathfrak{R}_i}{\mathfrak{F}}. \tag{8}$$

If $\mathfrak{F}$ is the value of $4\pi NI$ at which release occurs, the corresponding value of z given by (8) may be substituted in (6) to determine the release time. In this way there have been determined the values of $t\mathfrak{R}_i/(4\pi G)$ plotted against $\mathfrak{F}/(\mathfrak{R}_i(\varphi'' - \varphi_0))$ in Fig. 6. This is a universal curve for the relation between release time and the ampere turn value at which release occurs. The observed relation for any specific case is given by this curve, displaced vertically by the value of $4\pi G/\mathfrak{R}_i$ and horizontally by the value of $\mathfrak{R}_i(\varphi'' - \varphi_0)$ for the case in question. This is illustrated in Fig. 7, which shows the observed release time versus release ampere turn curves for the Y type relay for two sleeve sizes, corresponding to

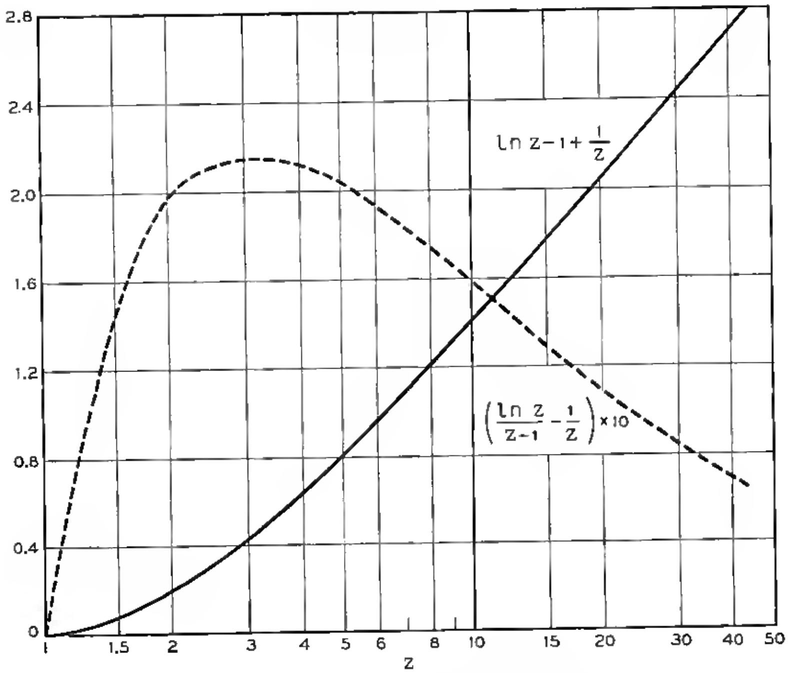

Fig. 5 — Factors for computing release time.

different values of G. For each sleeve size two curves are shown, corresponding to the limits of variation in magnetic characteristics, or in $\mathfrak{R}_i$ and $\varphi'' - \varphi_0$. The dotted curve included for comparison is the relation of Fig. 6.

An important property of the t versus $\mathfrak{F}$ relation can be demonstrated by substituting in (6) the expression for $\mathfrak{R}_i$ given by (8). There is thus obtained the equation:

$$t = \frac{4\pi G(\varphi'' - \varphi_0)}{\mathfrak{F}} \left(\frac{\ln z}{z - 1} - \frac{1}{z}\right). \tag{9}$$

A plot of the function of z appearing in brackets in this equation is included in Fig. 5. In the vicinity of the maximum at $z = 3.09$, this function is nearly a constant, varying little for values of z between 2 and 6. In this range t varies inversely with $\mathfrak{F}$, and the proportionality constant depends only upon G, the sleeve conductance, and the term $\varphi'' - \varphi_0$, which is relatively independent of material and dimensional variations, as shown later. This confirms the conclusion, previously indicated by graphical analysis, that through a considerable range of operation the

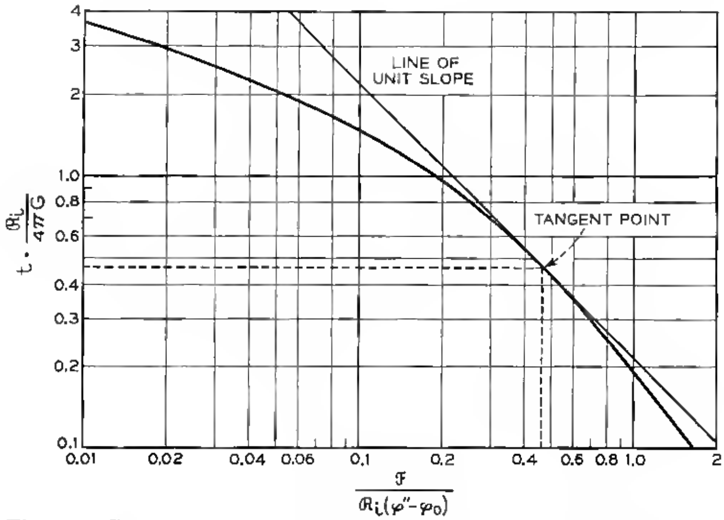

Fig. 6 — General relation between release time and ampere turns.

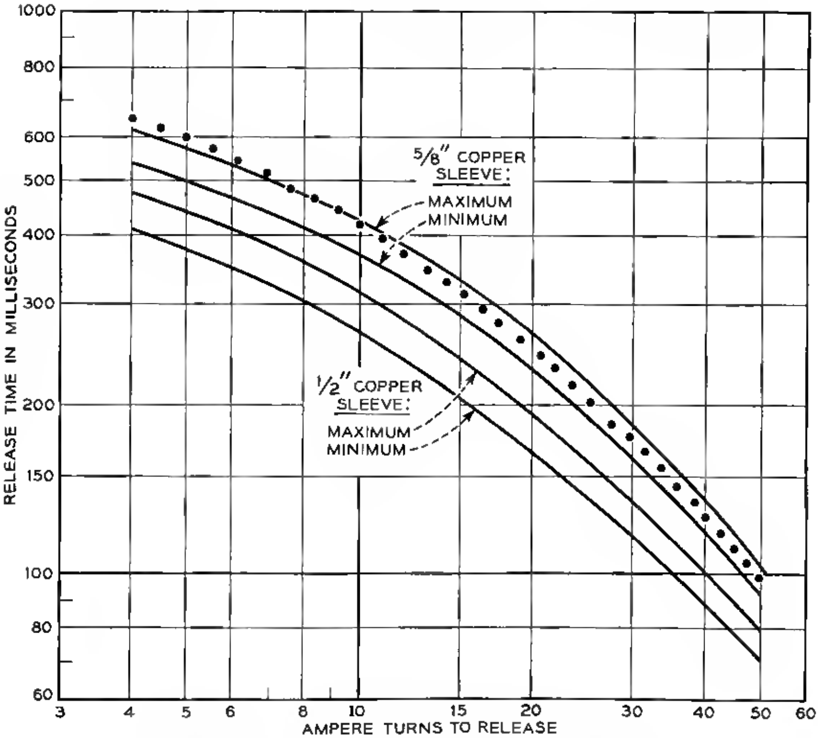

Fig. 7 — Observed release time characteristics.

release time is relatively independent of magnetic variations, provided it is adjusted to release at a specified ampere turn value.

The range in which t is inversely proportional to $\mathfrak{F}$ is that in which the logarithmic plot of Fig. 6 has a slope near unity. The point of tangency with a line of unit slope coincides with the value of z for which the bracketed function of z in (9) is a maximum. By equating the derivative of this to zero, it is found that this maximum occurs for $z = 3.09$, corresponding, from (8), to a value of 0.46 for $\mathfrak{F}/(\mathfrak{R}_i(\varphi'' - \varphi_0))$. From (6), the corresponding value of $t\mathfrak{R}_i/(4\pi G)$ is 0.46. Thus by drawing a line of unit slope tangent to the observed relation between t and $\mathfrak{F}$, and reading the co-ordinates of the point of tangency, $4\pi G/\mathfrak{R}_i$ and $\mathfrak{R}_i\,(\varphi'' - \varphi_0)$ may be evaluated. If G is known, these suffice to determine the values of $\mathfrak{R}_i$ and $\varphi'' - \varphi_0$.

Thus the release time is given by (6), and may be expressed in terms of the flux φ at which release occurs by means of (7), or in terms of the corresponding value of $\mathfrak{F}$, or $4\pi NI$, by means of (8). To relate the time to the spring load determining release, expressions are required relating the pull to φ or to $\mathfrak{F}$. To relate both time and pull characteristics to the design requires means for evaluating the magnetic constants and G in terms of the dimensions and materials of the design. The magnetic constants and the pull relations are discussed in the two following sections.

3 DECREASING MAGNETIZATION RELATIONS

To determine the decreasing magnetization relation experimentally, the magnet is demagnetized, and a measurement made of the flux developed on applying a full "soak," or high ampere turn value. The decreasing flux is then measured as the applied current is reduced, and finally reversed to determine $\mathfrak{F}_C$, the value required to restore the field to zero. The relation between φ and $\mathfrak{F}$ thus determined has the character shown in Fig. 1. If this curve conforms to the empirical relation given by (3) and (4), it is characterized by three constants: $\mathfrak{R}''$, φ'', and either $\mathfrak{F}_C$, as in (3), or φ_0, as in (4). These may be evaluated from measurements or estimated in preliminary design by the procedures indicated below.

Experimental Determination of Magnetic Constants

Equation (3) may be written in the form:

$$\frac{\mathfrak{F}_C + \mathfrak{F}}{\varphi} = \mathfrak{R}'' + \frac{\mathfrak{F}_C + \mathfrak{F}}{\varphi''}. \tag{10}$$

As $\mathfrak{F}_C$ may be read directly from the measured curve, as indicated in Fig. 1, values of $(\mathfrak{F}_C + \mathfrak{F})/\varphi$ may be computed from corresponding values of φ and $\mathfrak{F}$, and plotted against the corresponding value of $\mathfrak{F}_C + \mathfrak{F}$. One such plot for a slow release relay model is shown in Fig. 8. In agreement with (10), the plot is approximately linear over the range covered, and the slope and intercept may be used to evaluate $\mathfrak{R}''$ and φ'' as indicated in the figure. If co-ordinate paper is used, as in Fig. 8, with radial lines spaced to give a convenient scale of φ, the value of φ'' is that corresponding to the radial line parallel to the plot. The observed linearity of this relation does not extend to much higher values of $\mathfrak{F}$, and the observed asymptote φ'' is fictitious, as the full curve is concave downwards, asymptotic to a lower value of φ than that found in this way. If the observed value of φ_0 does not agree with that computed from values of the other constants, it is preferable to use the observed rather thad the computed value of φ_0.

As shown in the preceding section, values of $\mathfrak{R}_i$ (which is related to $\mathfrak{R}''$ by (5)) and of $\varphi'' - \varphi_0$ may be independently determined from measurements of release time versus release ampere turns. In cases of disagreement, these values are to be preferred to those determined from the decreasing magnetization measurements. The latter are primarily of interest in checking design estimates, and in indicating the effects of variations in dimensions and material properties.

Estimation of Magnetic Constants

In the "tight" magnetic circuit of a slow release relay, tbe leakage field may be ignored, and the reluctance taken as the sum of the iron reluctance and that of the air gaps, which may be designated $\mathfrak{R}_E$. The estimation of $\mathfrak{R}_E$ is discussed in the following section, in connection with the pull relations. The iron reluctance is substantially that of a permanent magnet, supplying flux to the external circuit of reluctance $\mathfrak{R}_E$. In using (3) to characterize the magnetization relation, the demagnetization in the second quadrant is taken as continuous with the decreasing magnetization in the first quadrant. Thus the value of $\mathfrak{F}_C$ is that which would apply to a permanent magnet of length ℓ, equal to that of the iron path, as given by:

$$\mathfrak{F}_C = H_C \ell, \tag{11}$$

where H_C is the coercive force of the material. For the soft magnetic materials used for slow release relays, H_C is of the order of 0.5 to 1.0 oersteds, so if $\ell = 10$ cm, $\mathfrak{F}_C$ lies in the range from 5 to 10 gilberts (4 to 8 ampere turns).

Similarly, the initial iron reluctance corresponds to the slope of the demagnetization curve at H_C, as given by the initial permeability μ''. Thus the total initial reluctance $\mathcal{R}''$ is given by the equation:

$$\mathcal{R}'' = \mathcal{R}_E + \frac{1}{\mu''} \sum \frac{\ell}{a}, \tag{12}$$

where $\sum \ell/a$ denotes the sum of terms corresponding to the component parts of the iron path, each term giving the length of the part divided by its cross sectional area. As μ'' is of the order of 20,000 for soft magnetic materials, the iron reluctance term is small compared with $\mathcal{R}_E$, which is typically of the order of 0.010 cm^{-1} or less.

The value of φ'' may be estimated as the saturation flux for the core, or $B''a$, where a is the cross sectional area of the core. B'' is the saturation density, which may be taken as the value of B_M listed in a companion article.[6] Estimates of φ'' thus obtained are lower than those which best fit the decreasing magnetization measurements. No convenient means for correcting this disparity has been found, other than an arbitrary increase by a factor of 20 per cent, based upon experience. In a particular relay design, however, the observed values of φ'' vary directly with the core cross section.

4 PULL RELATIONS

As shown by equation (6), the release time varies inversely as the incremental reluctance $\mathcal{R}_i$. This is proportional to $\mathcal{R}''$, in which the dominant

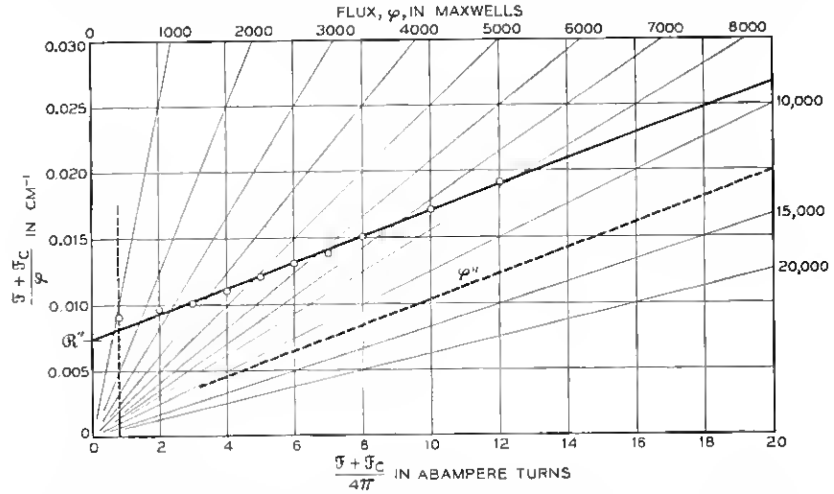

Fig. 8 — Evaluation of magnetic constants from measurements of decreasing magnetization.

term, from (12), is the total air gap reluctance $\mathcal{R}_E$. To obtain long delays, therefore, $\mathcal{R}_E$ must be made small, and consequently sensitive to small dimensional variations. These may be compensated in an initial adjustment, but subsequent changes must be minimized if constancy of performance is to be attained.

Two expedients have been used to provide a small and stable gap reluctance. The older one is the use of a "residual screw," an adjustable non-magnetic member which serves as an armature stop and assures a small air gap at the pole face. With this scheme, the residual screw is used to adjust the relay. The alternative scheme, used in the flat type relays of the Bell System, is to employ a domed pole face on the armature, providing a spherical surface in contact with a mating plane surface on the core. The only effective air gap at the point of contact is that of the chrome finish on the parts. With this scheme, the relay is adjusted by varying the spring tension, and thus the operated load.

General expressions for the pull of electromagnets are discussed in a companion article,[6] where it is shown that the pull F provided by a gap flux φ is given by the equation:

$$F = \frac{\varphi^2}{8\pi} \frac{d\mathcal{R}_G}{dx}, \tag{13}$$

where x is the dimension in the direction of the pull, and $\mathcal{R}_G$ is the gap reluctance. In the usual case, $\mathcal{R}_G$ varies linearly with x, and $d\mathcal{R}_G/dx$ has the constant value $1/A$, where A is the effective pole face area. This is applicable to any case of plane mating surfaces having an appreciable separation, including the configuration usually employed with a residual screw as separator.

Pull for a Domed Pole Face

In the case of a domed pole face there is a concentration of the field near the point of contact, which varies with the effective air gap at this point. An expression for the reluctance can be developed for the idealized configuration shown in Fig. 9. In this, R is the radius of a spherical surface mating with a plane over the projected area A bounded by the radius αR. The separation x is that measured at the center of A. As indicated in the figure, an expression can be obtained for the gap reluctance $\mathcal{R}_G$ in terms of its reciprocal, or permeance. The latter is given by the integration of the permeances of the differential rings within the projected area. $\mathcal{R}_G$ is conveniently expressed in terms of the ratio $\mathcal{R}_\infty/\mathcal{R}_G$

given by the equation:

$$\frac{\mathcal{R}_\infty}{\mathcal{R}_G} = 2\pi R\mathcal{R}_\infty \ln\left(1 + \frac{1}{2\pi R\mathcal{R}_\infty}\right), \tag{14}$$

in which $\mathcal{R}_\infty = x/A$, the value of $\mathcal{R}_G$ which would apply if the spherical radius were infinite, and both mating areas plane. On substituting this expression for $\mathcal{R}_G$ in (13), there is obtained the following equation for the pull at a domed pole face:

$$F = \frac{\varphi^2}{8\pi A}\left(\frac{\mathcal{R}_G}{\mathcal{R}_\infty}\right)^2 \frac{2\pi R\mathcal{R}_\infty}{1 + 2\pi R\mathcal{R}_\infty}. \tag{15}$$

The right hand side of (14), and the expression given by (15) for the ratio of F to $\varphi^2/(8\pi A)$ are both functions of a single dimensionless parameter: $R\mathcal{R}_\infty$. These two ratios are plotted against this parameter in Fig. 10. These curves show how the reluctance and pull values compare

For $\theta < \alpha$ small,

$$r = R\sin\theta = R\theta$$

$$h = R(1 - \cos\theta) = \frac{R\theta^2}{2}$$

Gap Reluctance

$$\frac{1}{\mathcal{R}_G} = \int_0^{\alpha R} \frac{2\pi r \times dr}{x + h}$$

$$= \int_0^{\alpha} \frac{2\pi R^2\theta d\theta}{x + \dfrac{R\theta^2}{2}}$$

$$= 2\pi R \ln\left(1 + \frac{R\alpha^2}{2x}\right)$$

Let: $\mathcal{R}_\infty = \dfrac{x}{A} = \dfrac{x}{\pi(\alpha R)^2}$

$$\frac{\mathcal{R}_\infty}{\mathcal{R}_G} = 2\pi R\mathcal{R}_\infty ln\left(1 + \frac{1}{2\pi R\mathcal{R}_\infty}\right)$$

Pull for Flux φ

$$F = \frac{\varphi^2}{8\pi} \times \frac{d\mathcal{R}_G}{dx} = \frac{\varphi^2}{8\pi A} \times \frac{d\mathcal{R}_G}{d\mathcal{R}_\infty}$$

$$= \frac{\varphi^2}{8\pi A} \times \left(\frac{\mathcal{R}_G}{\mathcal{R}_\infty}\right)^2 \times \frac{2\pi R\mathcal{R}_\infty}{1 + 2\pi R\mathcal{R}_\infty}$$

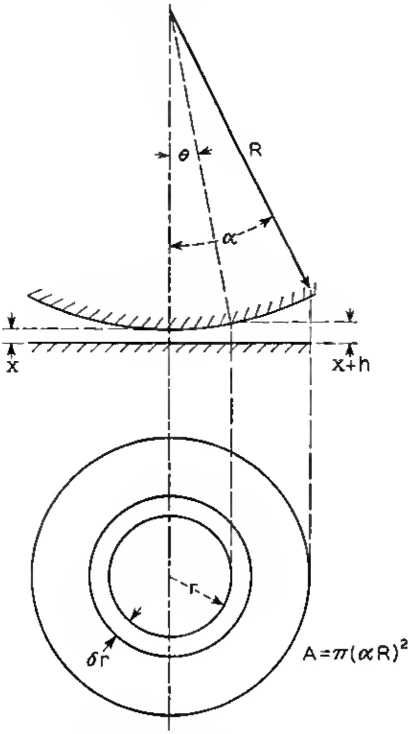

Fig. 9 — Reluctance and pull of a domed pole face.

with those for a gap of the same separation and projected area, but with plane pole faces (a dome of infinite radius).

Ampere Turn Sensitivity

The effect of the relation between F and φ on the release time cannot be conveniently deduced from (6), which involves both the term in z, which varies with φ, and the reluctance $\mathcal{R}_i$, which also enters the pull relation. If the pull is expressed in terms of $\mathfrak{F}$, however, (9) shows that the time varies inversely with $\mathfrak{F}$, and is substantially independent of the reluctance, provided z is in the usual range from 2 to 6. It follows that maximum release time can be obtained for a given load if the value of $\mathfrak{F}$

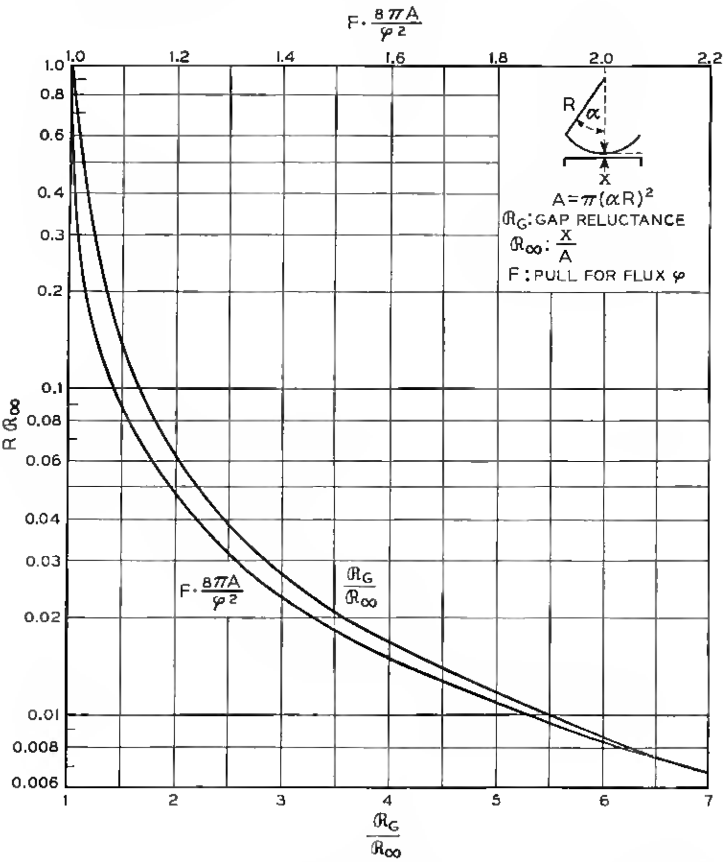

Fig. 10 — Reluctance and pull relations of a domed pole face electromagnet.

required to operate this load is minimized. Thus maximum release time is attained by providing maximum ampere turn sensitivity.

A general expression for the relation between F and $\mathfrak{F}$ may be obtained by substituting in (13) the expression for $(\mathfrak{F}_C + \mathfrak{F})/\varphi$ given by (10). A much simpler relation is given by the linear approximation to the decreasing magnetization curve in which $(\mathfrak{F}_C + \mathfrak{F})/\varphi$ is taken as equal to $\mathfrak{R}_i$. This approximation has the same slope at φ_0 as the curve given by (3) or (10). The release pull usually corresponds to values of φ much nearer φ_0 than φ'', and the approximation is satisfactory in this range if φ_0/φ'' is small. Writing $(\mathfrak{F}_C + \mathfrak{F})/\mathfrak{R}_i$ for φ in (13), there is obtained the equation:

$$F = \frac{2\pi(NI + (NI)_C)^2}{\mathfrak{R}_i^2} \frac{d\mathfrak{R}_G}{dx}, \tag{16}$$

in which $(NI)_C$ is written for $\mathfrak{F}_C/(4\pi)$. This approximation is used for convenience and simplicity. The general expression, which is required for higher values of $\mathfrak{F}$, is obtained by substituting the right hand side of (10) for $\mathfrak{R}_i$ in (16) and in the expressions derived from it.

By comparison with (12), $\mathfrak{R}_i$ may be taken as equal to the gap and joint reluctance $\mathfrak{R}_E$ plus a modified and minor term for the iron reluctance. For the present purpose, it is convenient to take $\mathfrak{R}_i$ as the sum of the main gap reluctance $\mathfrak{R}_G$, which varies with x, and $\mathfrak{R}_F$, which includes the iron reluctance and the constant reluctances of the heel gap and of any joints in the magnetic structure. For a plane pole face, $\mathfrak{R}_G$ is given by x/A, where A is the effective pole face area, and (16) reduces to the equation:

$$F = \frac{2\pi(NI + (NI)_C)^2}{A\left(\mathfrak{R}_F + \frac{x}{A}\right)^2}.$$

As shown in one of the companion articles,[6] the pull F at travel x for a given value of NI is a maximum when the gap reluctance x/A is equal to the reluctance $\mathfrak{R}_F$ external to the gap. A similar relation applies for a domed pole face gap, as can be shown from the expressions given above. Taking the linear approximation to apply, $(\mathfrak{F}_C + \mathfrak{F})/(\mathfrak{R}_F + \mathfrak{R}_G)$ may be substituted for φ in (15). Writing $x/\mathfrak{R}_\infty$ for A in the resulting equation, this may be written in the form:

$$F = \frac{2\pi(NI + (NI)_C)^2}{x\mathfrak{R}_F} \Omega,$$

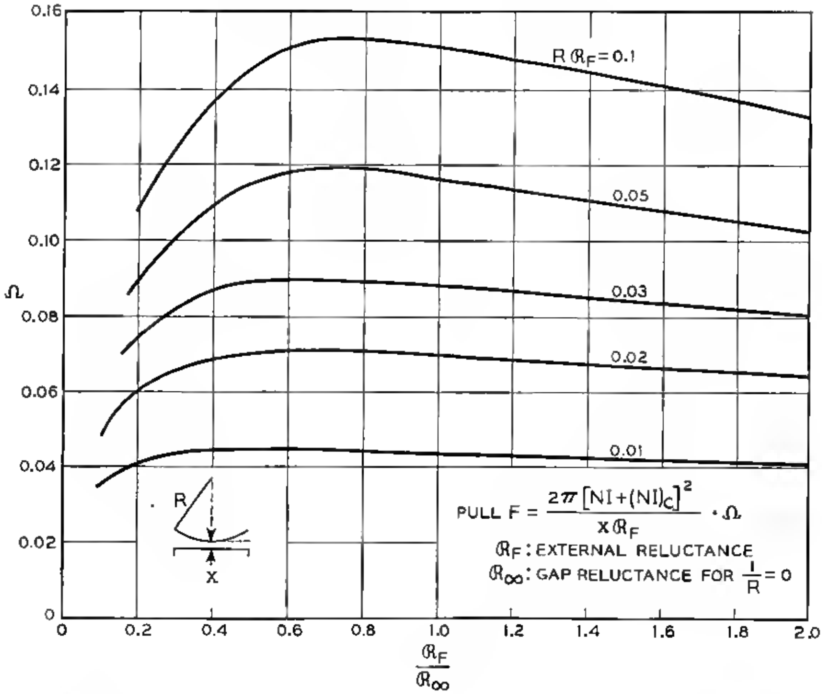

Fig. 11 — Ampere turn sensitivity of a domed pole face electromagnet.

where: (17)

$$\Omega = \frac{\mathcal{R}_G^2 \mathcal{R}_F}{\mathcal{R}_\infty(\mathcal{R}_G + \mathcal{R}_F)^2} \frac{2\pi R \mathcal{R}_\infty}{1 + 2\pi R \mathcal{R}_\infty}.$$

The ratio Ω is a function of $\mathcal{R}_F/\mathcal{R}_\infty$ and of $R\mathcal{R}_\infty$, which determines $\mathcal{R}_G/\mathcal{R}_\infty$, as shown by (14). Alternatively, Ω may be taken as a function of the ratios $\mathcal{R}_F/\mathcal{R}_\infty$ and $R\mathcal{R}_F$, and Ω may be represented, as in Fig. 11, by a family of curves giving Ω versus $\mathcal{R}_F/\mathcal{R}_\infty$ for various values of $R\mathcal{R}_F$.

From (17), maximum ampere turn sensitivity is attained by minimizing the separation x and the reluctance $\mathcal{R}_F$ external to the gap. Assuming these to be made as small as engineering considerations permit, the pull for a given value of NI varies as Ω. With x and $\mathcal{R}_F$ fixed, Ω now depends only on the dome radius R, to which $R\mathcal{R}_F$ is now proportional, and on the projected area A, to which $\mathcal{R}_F/\mathcal{R}_\infty$ is now proportional. From the curves of Fig. 11 it is apparent that maximum ampere turn sensitivity is attained by using as large a value of dome radius as possible. For a given dome radius, there is an optimum value of $\mathcal{R}_F/\mathcal{R}_\infty$,

and hence an optimum value of A, corresponding to the maximum shown by each curve in Fig. 11.

AG Relay Pull

The AG relay[3] is a slow release relay with a domed pole face, to which the relations given above apply approximately. M. A. Logan and O. C. Worley developed a more exact expression for the pull in this case, in which an increment to the pull is given by the secondary pole faces on the side legs of the armature, which mate with the side legs of the E shaped core member. A further increment to the pull is given by the area at the main gap which lies outside the dome proper. Their analysis may be summarized in the notation used here by writing:

$$\mathcal{R}_i - \mathcal{R}_I = \mathcal{R}_S + \frac{\mathcal{R}_G \mathcal{R}_P}{\mathcal{R}_G + \mathcal{R}_P},$$

where R_I is the iron reluctance, $\mathcal{R}_S$ is the side gap reluctance, and the main gap reluctance is that of the domed gap $\mathcal{R}_G$ in parallel with that of the remaining area, $\mathcal{R}_P$. Substituting $R_i - \mathcal{R}_I$ for $\mathcal{R}_G$ in (16), there is obtained the following expression for the pull:

$$F = \frac{2\pi(NI + (NI)_C)^2}{\mathcal{R}_i^2}\left(\frac{d\mathcal{R}_S}{dx} + \left(\frac{\mathcal{R}_G}{\mathcal{R}_G + \mathcal{R}_P}\right)^2 \frac{d\mathcal{R}_P}{dx} + \left(\frac{\mathcal{R}_P}{\mathcal{R}_G + \mathcal{R}_P}\right)^2 \frac{d\mathcal{R}_G}{dx}\right). \tag{18}$$

An expression for $d\mathcal{R}_G/dx$ is given in Fig. 9, while $d\mathcal{R}_S/dx$ and $d\mathcal{R}_P/dx$ are given by $1/A_S$ and $1/A_P$, where A_S and A_P are the effective pole face areas for these two gaps. For the dimensions applying to the AG relay, these additional terms introduce minor but significant corrections to the values of F computed from (17).

Engineering Pull Data

In determining the requirements for slow release relays, the estimation of the range of variation in pull is a major problem. A procedure for such estimation developed by M. A. Logan makes effective use of the relation between F and $(NI + (NI)_C)$ in (16). This relation gives a linear plot of slope two when F is plotted against $(NI + (NI)_C)$ on logarithmic paper. A basic plot of this nature may be experimentally determined for a model having nominal values of coercive force H_C, to which $(NI)_C$ is proportional, and of finish thickness and side gap separa-

tion. The effect of known changes in these two latter factors may be determined by the observed vertical shift in this logarithmic plot, corresponding to changes in $\mathcal{R}_i$ and $d\mathcal{R}_G/dx$. The effect of a change in H_C is a change in the value of $(NI)_C$ to be subtracted from $(NI + (NI)_C)$ in determining NI. From these results there could be prepared curves giving the relation between F and NI for different combinations of the several variables for which allowance should be made.

5 EVALUATION OF CONDUCTANCE

The preceding sections have described procedures for estimating and measuring all the terms entering the expressions for the release time except the conductance G. This quantity is the sum of the sleeve conductance G_S and the eddy current conductance G_E. In some cases a short circuited winding may be used instead of a sleeve: in such cases G is the sum of G_E and the coil constant of the winding, G_C or N^2/R.

Sleeve Conductance

The conductance of a cylindrical sleeve is the sum of the conductances of the differential shells of length ℓ, radius r and thickness δr, as indi-

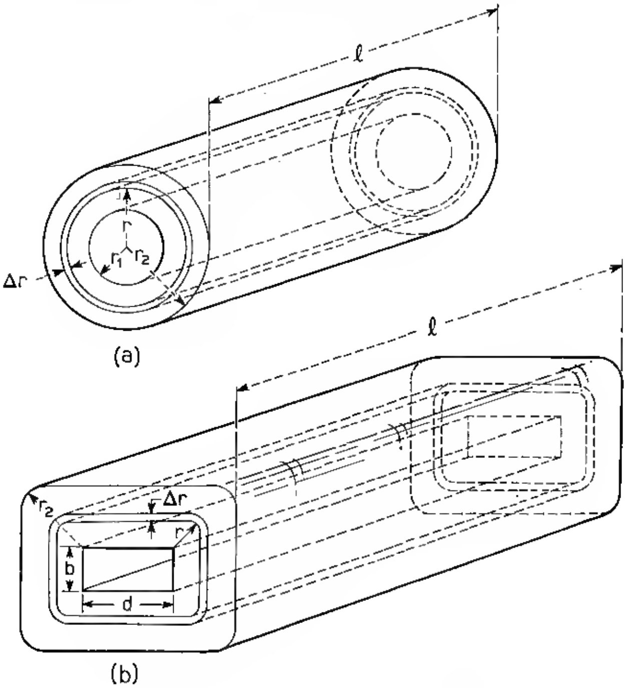

Fig. 12 — Sleeve conductance relations.

cated in Fig. 12(a). Thus the total conductance G_S is given by:

$$G_S = \int_{r_1}^{r_2} \frac{\ell dr}{2\pi\rho r},$$

where ρ is the resistivity of the material. On integration, there is obtained:

$$G_S = \frac{\ell}{2\pi\rho} \ln \frac{r_2}{r_1}. \tag{19}$$

The value of ρ for copper is 1.73×10^{-6} ohm-cm. If r_2 is twice r_1, for example, and $\ell = 5$ cm, the value of G_S given by (19) for a copper sleeve is 320,000 mhos.

In the case of a sleeve of rectangular section, as shown in Fig. 12(b), an approximation may be obtained by taking the sleeve as made up of shells with straight sides parallel to the center hole, connected by quarter circles. Then the perimeter of the shell at a distance r from the center hole is $2(b + d + \pi r)$. The total sleeve conductance is therefore given by:

$$G_S = \int_0^{r_2} \frac{\ell\, dr}{2(b + d + \pi r)\rho}.$$

in which r_2 is the wall thickness. On integration, there is obtained the equation:

$$G_S = \frac{\ell}{2\pi\rho} \ln \left(\frac{b + d + \pi r_2}{b + d}\right). \tag{20}$$

This expression is identical with that for the cylindrical sleeve, as given by (19), when the ratio r_2/r_1 of the radii is equal to the ratio of $(b + d + \pi r_2)$ to $(b + d)$.

Coil Conductance

For a cylindrical coil, the number of turns N is determined by the area of the coil section cut by a plane through the axis, or $\ell(r_2 - r_1)$, where ℓ is the length of the coil and r_1 and r_2 are its inner and outer radii. If a is the cross sectional area of the wire, and e the fraction of the coil space occupied by the conductor,

$$Na = e\ell(r_2 - r_1).$$

The mean length of turn is $\pi(r_2 + r_1)$, and the total length of conductor is N times this. Hence the resistance R is given by:

$$R = \frac{\rho N\pi(r_2 + r_1)}{a}.$$

The expression for N/R given by these equations gives the following expression for the coil constant N^2/R, or G_C :

$$G_C = \frac{e\ell}{\pi\rho} \frac{r_2 - r_1}{r_2 + r_1}. \tag{21}$$

For unit length of coil, values of G_C are shown in Fig. 13, plotted against r_2/r_1 for various values of e, together with the corresponding relation for G_S, as given by (19). This shows that the sleeve provides the maximum value of conductance for the space occupied. It also shows that the space used for a given value of sleeve conductance reduces the value of G_C that can be provided. When the full depth of the winding space is used by both coil and sleeve, called a slug in this case, each occupying part of the length, the value of G_C attainable is reduced in proportion to the length used by the slug. When the coil is outside the sleeve, the outer radius of the sleeve is the inner radius of the coil, and the value of G_C is reduced in proportion to the value of G_S. For a given value of e (given wire size and insulation) the value of G_C is fixed by the winding space and the value of G_S.

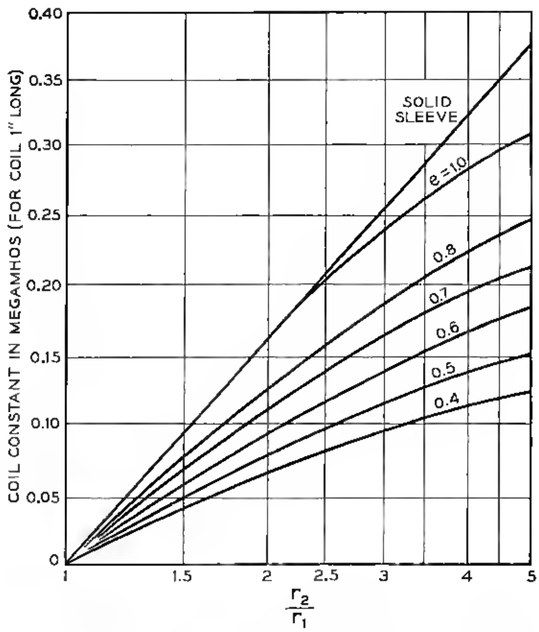

Fig. 13 — Relation between coil constant and coil dimensions.

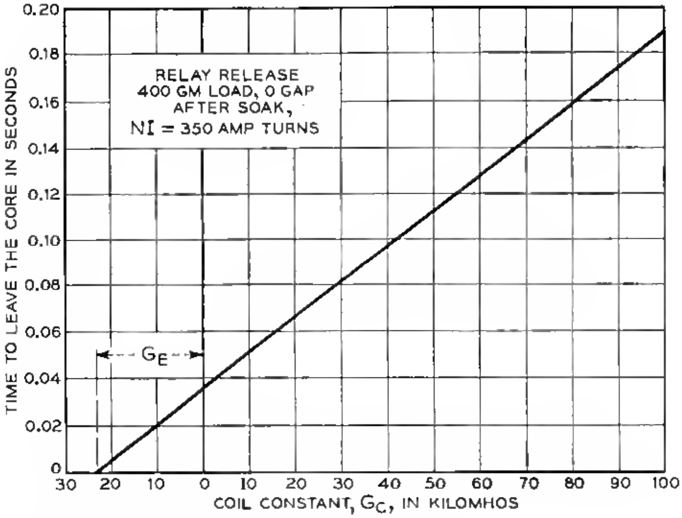

Fig. 14 — Experimental evaluation of eddy current conductance for release of relay.

Eddy Current Conductance

In one of the companion articles[4] it is shown that when the eddy current conductance G_E is a minor term in G, as with slow release relays, it is given by the equation:

$$G_E = \frac{\ell}{8\pi\rho}, \tag{22}$$

where ρ is the resistivity of the material, and ℓ is the length of the magnetic path. For iron, $\rho = 11 \times 10^{-6}$ ohm-cm, so for $\ell = 5$ cm, the value of G_E given by (21) is 17×10^3 mhos. The equation applies to a path of uniform circular cross section, so that the effective value of ℓ for most relay structures is intermediate between that of the core and that for the complete path.

The eddy current conductance of a specific model may be experimentally determined by measuring the release time with a winding shorted through an external resistance. A series of measurements are made in which this resistance is varied, while the initial current, which determines the "soak NI" value, is kept constant. The different values of the resistance correspond to different values of the coil constant G_C or N^2/R. From (6), the time t in this series of measurements varies directly as G, or $G_C + G_E$. Then a plot of t versus G_C, as illustrated in Fig. 14, is linear, and has a negative intercept giving the value of G_E.

Values of G_B thus determined agree with those estimated from (22) within the level of uncertainty as to the applicable value of ℓ in this equation.

6 DESIGN OF SLOW RELEASE RELAYS

The following discussion is confined to the bearing on design decisions of the performance relations developed in the preceding sections, and does not cover the manufacturing considerations involved. The development of a specific design depends on the initial choice between certain alternative features which require description.

Design Alternatives

The features considered here are: (1) the adjustment means, (2) the form of sleeve, (3) the criterion of adjustment.

The two methods of adjustment that have been used are (a) residual screw adjustment, (b) spring load adjustment. The former affects the release time by changing the reluctance and the residual flux, the latter by changing the flux or ampere turn value at which release occurs. The differences in the two methods relate more to the stability of the adjustment than to its ease or initial accuracy. Spring adjustment permits the use of the domed pole face, which is inherently stable except as the finish thickness may be affected by wear.

The two forms of sleeve are the interior sleeve, which uses the full length of the winding space, but only part of the depth, and the slug, which uses the full depth, but only part of the length. As shown in Section 5, the values of G_S and G_C, or coil constant N^2/R, attainable with a given winding space are independent of which arrangement is used. The slug provides some flexibility as to operate time, on which its retarding effect is a maximum when it is near the gap, and a minimum when it is away from it. It is subject to smaller temperature changes, with consequent changes in conductance, than the sleeve. The other differences between the two arrangements relate to manufacture, and to the costs of both coil and sleeve. The interior sleeve is used with the flat type relays of the Bell System.

The two different criteria of adjustment used are the release current, and the release time. The former is an indirect control, using a measurement of release ampere turns to determine the time that will result for the sleeve conductance used: the other is a direct measurement of the quantity to be controlled. In principle, the latter method would appear preferable, but its use is attended with several disadvantages.

The most important of these is the uncertainty as to the sleeve temperature that applies to any measurement made in central office maintenance, unless the relay to be tested is cut out of service for an hour or more before measurement. The conductivity of copper varies approximately as its absolute temperature, or, for engineering estimates, as $390 + T_F$, where T_F is the temperature in degrees Fahrenheit. Coil temperatures of 225°F are permitted in normal relay operation. As the time varies as the sleeve conductance, a relay with its sleeve at this temperature would have a release time in the ratio 470/615 or 76/100 to the rated release time for 80°F. Allowance for variation in this range is made in circuit design, but a corresponding uncertainty as to the condition applying in adjustment would effectively double this variation. Current flow adjustment is free of this difficulty, and the variations in the correlation of release time with release ampere turns are less than those resulting from the temperature uncertainty in any convenient procedure for timing measurements. Current flow adjustment has the further advantage of using equipment that is employed for other relays in central office maintenance. It is the more commonly used criterion of adjustment for Bell System relays.

Operate Considerations

A slow release relay must not only provide the desired release performance: it must also operate its load. The operate pull characteristics are similar to those of other relays of the same general type, as the domed pole face, in particular, gives nearly the same pull at an open gap as a plane pole face of the same total area. Thus the pull characteristics of the *AG* relay are similar to those of the *AJ* relay[3] for the same travel. The sleeve retards the flux development, and makes operation slower than that for the same coil input without the sleeve. In most applications of slow release relays, this has little or no effect on circuit operation. Faster operation can be obtained by increasing the steady state power applied, but this is limited by heating considerations. The large part of the winding space used for the sleeve limits the operate sensitivity of slow release relays, and increases the power required for a given load. The load for a given relay design determines a minimum ampere turn value for operation, and this is related to the steady state power by the identity: $(NI)^2 = I^2R\, N^2/R$. As the coil constant N^2/R, or G_C, is determined by the available winding space available for the coil, the power requirements of slow release relays are higher than those of similar relays having the full winding space available for the coil.

Optimum Design Conditions

In considering the design features that are favorable to slow release operation it is convenient to refer again to the expression for the release time given by equation (9):

$$t = \frac{4\pi G(\varphi'' - \varphi_0)}{4\pi NI} f(z), \tag{9}$$

where $f(z)$ is the function shown in Fig. 5 to be nearly a constant in the range of interest. It is also convenient to refer to the expression for the pull given by the equation (16):

$$F = \frac{2\pi(NI + (NI)_C)^2}{\mathcal{R}_i^2} \frac{d\mathcal{R}_G}{dx}.$$

Together with reference to the operate requirements, these two equations indicate the characteristics that are important for slow release operation.

The winding space determines the value of G attainable, as shown by the relations of Section 5. It limits the combined values of G_S and G_C, of which the latter controls the operate power sensitivity, while the former, from (9), determines the release time. Thus both the operate sensitivity and the release time attainable vary directly with the winding space and hence with the over-all size of the relay.

From (9), the attainable release time is nearly proportional to φ'', and hence to the cross section of the core, assuming φ_0/φ'' to be small. The external core dimension is the internal dimension of the sleeve, so that increases in φ'' are offset by decreases in G if this dimension alone is varied. In any case, sufficient section must be provided for φ'' to have a margin over the field required to operate the maximum load.

In telephone use, the slow release relays are a minority group in a relay population which must, for maximum economy in manufacture and use, have common overall dimensions and as few differences as are consistent with the requirements of specific uses. Thus the core section, winding space, and over-all dimensions reflect an optimum choice for the whole relay population, and not for the slow release relays alone. The latter are distinguished by as few special features as are essential to their special function. In the *AG* relay these are: the armature, the heat treatment of the magnetic parts, the sleeve and coil, and a buffer spring for load adjustment.

The material and its heat treatment determine the iron reluctance and the coercive mmf, $4\pi(NI)_C$, of which the latter is the more important quantity. It enters the timing relation indirectly in the minor term φ_0,

which varies directly with $(NI)_C$, and enters the pull relation (16) directly as a major term. If high coercive material were used to increase $(NI)_C$ and thus reduce the release ampere turns NI, the effect of the latter on the time would be offset by the increase in φ_0. The latter varies also with the reluctance $\mathcal{R}_t$, and if φ_0/φ'' is not small, the time for a given value of NI is no longer independent of the reluctance variation, and the release ampere turn value is unsatisfactory as a criterion of adjustment. Thus a small stable value of $(NI)_C$, as given by low coercive material, is preferable when the release current is used as the criterion of adjustment.

The pole face dimensions are determined by the need for a low stable value of gap reluctance. As shown in Section 4, maximum release sensitivity is obtained by using as large a dome radius as will assure a consistent configuration, together with a projected area optimum with respect to the reluctance in series with the gap. Fig. 11 shows this optimum to be broad, allowing considerable latitude in the choice of the projected area with reference to other design considerations.

The finish applied to the magnetic parts at the pole face is critical with respect to wear and stability, as well as to the danger of mechanical adhesion which would affect the release load. A nickel chrome finish is used in the Y and AG relays. The effective air gap thus introduced is the dimension x of Figs. 9 and 10. As these figures show, this has a relatively large effect on the pull relation, and any change in this dimension as a result of wear tends to increase the pull and the release time. The analysis of Section 4 shows that maximum sensitivity is attained with as small a finish separation as can be provided, which is fortunately in agreement with the manufacturing considerations for the finish.

The adjustment means used with a fixed pole face, e.g., the dome type, is some form of spring adjustment for controlling the operated load. The normal operated load of a relay is the product of the contact force and the number of contacts, plus the back tension and the force required for spring flexure. Adjustment of the back tension is used, but is limited by the requirements for operation. Hence a buffer spring is employed in AG relays, which is picked up in the last few mils of armature travel and affords control of the operated load. This spring is adjusted by bending, and a tolerance range of 50 gm wt is allowed in this adjustment.

Performance Variations

Performance variations fall into two categories: the initial differences among individual relays that are nominally identical, and the further differences that develop in service as a result of wear and other changes.

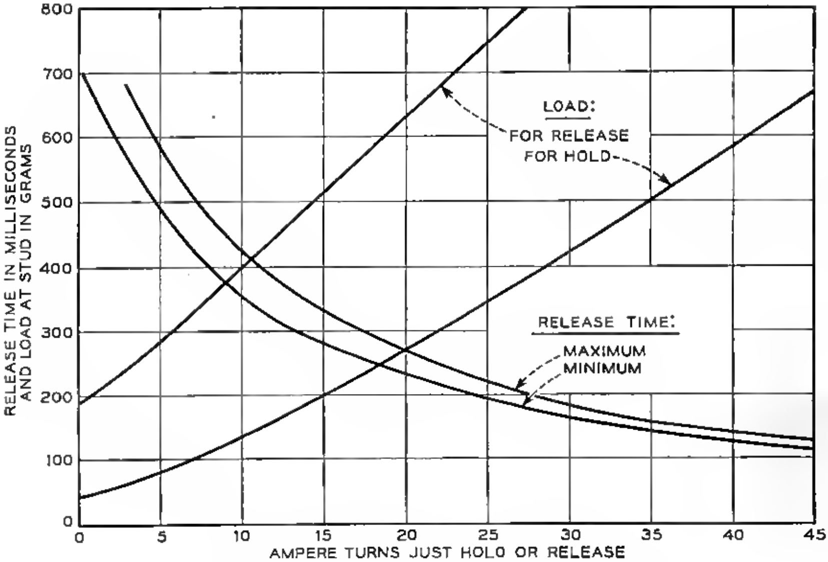

Fig. 15 — Engineering data for release time estimation.

load attainable in all cases. This corresponds to the case in which the contact force, which is not subject to adjustment, has its maximum value. The NI value read from the hold curve for this load is the lowest that can be specified as a "hold" requirement. Each individual relay has a release F versus NI characteristic intermediate between the release and hold capability curves. To meet the hold requirement its load must be adjusted, and this adjustment is subject to the tolerance cited above. The minimum load is set by the hold requirement, and the maximum load is set by a release requirement of a lower NI value, the difference between the two NI requirements corresponding to the tolerance range in load adjustment.

Thus adjustment values are determined in the form of limits to the ampere turn value at which release occurs: the lower limit is the release value, the upper the hold. The release time limits can then be read from the timing curves. The maximum time is read from the maximum curve at the release ampere turn value. The minimum time is read from the minimum curve at the hold ampere turn value. This minimum time, when determined for the largest sleeve, is the longest time that can be guaranteed for the load in question, and is subject to reduction in service by the temperature variation previously discussed. When a shorter release time is desired, a smaller sleeve may be used, or a higher hold

value specified, subject to the capacity of the adjustment springs to supply the necessary increase in load.

In the common case for which only the minimum time is of circuit importance, the release value is chosen without reference to the hold value, solely for the purpose of assuring that the relay will not lock up indefinitely. This procedure takes advantage of the simpler requirements of this case by widening the adjustment tolerance and reducing adjustment effort.

7 CONCLUSIONS

The relation between the release time of slow release relays and the design parameters can be more accurately expressed in analytical form than the other time characteristics of relays. These analytical relations, as presented in this article, can be used for the estimation of release time, and particularly for the determination of the effect on this time of variations in the design parameters. The need for a low reluctance magnetic circuit makes the performance of slow release relays highly sensitive to dimensional and material variations, and adjustment is required to assure the timing limits required in their use. Such use usually permits a wide spread in release time, provided a minimum value is assured. Advantage is taken of this in providing slow release relays which perform their function at a minimum cost in manufacture and use, materially lower than that for the construction and adjustment practices which would be required for closer timing control.

ACKNOWLEDGEMENTS

The specific references made to individuals and prior studies do not include all the work which has been drawn on in the preparation of this article. In particular, much of the discussion of the applications of the analysis is based on work carried out by M. A. Logan, Mrs. K. R. Randall, O. C. Worley and others in the development of the *AG* relay.

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